

## Problem Set 3.1

Due Nov, 9th

### Q1. Production Technology

Consider a Cobb-Douglas production function  $Q = F(K, L) = K^a L^b$  with  $a > 0$ ,  $b > 0$ , and  $a + b = 1$ , where  $Q$  is the output level and  $K$ ,  $L$  is capital input and labor input level, respectively.

- (a) Verify that this production function shows constant returns to scale.
- (b) Calculate the output elasticity of labor  $\mu_L$  and capital  $\mu_K$ . Do they depend on the specific factor input levels?
- (c) Show that the marginal rate of technical substitution (MRTS) between labor and capital only depends on factor input ratio.
- (d) How will MRTS change when factor input ratio increases?

### Q2. Profit Maximization

Consider a general Cobb-Douglas production function  $Q = F(K, L) = K^a L^b$  with  $a > 0$ ,  $b > 0$ .

- (a) Under what conditions (restrictions on  $a$  and  $b$ ) is this production function concave? Now consider the case when  $a = b = 1/4$ . Suppose that the firm is a price taker in both factor markets and product market. Price of the product is  $p$ , price of capital is  $r$ , price of labor is  $w$ . Calculate the following:
  - (b) The optimal capital-labor input ratio.
  - (c) The optimal capital input  $K^*$ .
  - (d) How would optimal capital input change as  $r$  increases,  $w$  increases, or  $p$  increases? (Sign  $\frac{dK^*}{dr}$ ,  $\frac{dK^*}{dw}$ , and  $\frac{dK^*}{dp}$ )

### Problem Set 2.1, Q1 revised edition

Suppose that a person has utility function of the form:

$$U = y + 2l^{1/2}$$

where  $y$  is consumption, and  $l$  is leisure. The maximum leisure that can be consumed is 24 hours. The wage rate is  $w$ , and the price of consumption is  $p$ .

- a) Derive the individual's labor supply function.
- b) What is the minimum wage at which she is willing to work?
- c) How will her supply of hours respond to a small cut in a proportional income tax?

d) How will her supply of hours respond to a small cut in a proportional consumption tax?

(For part c) and d), **just answering "increase" or "decrease" is enough.** )